

Pumping through a quantum dot in the proximity of a superconductor

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(Dated: February 6, 2008)

We study adiabatic pumping through a quantum dot tunnel-coupled to one normal and one superconducting lead. We generalize a formula which relates the pumped charge through a quantum dot with Coulomb interaction to the instantaneous local Green's function of the dot, to systems containing a superconducting lead. First, we apply this formula to the case of a non-interacting, single-level quantum dot in different temperature regimes and for different parameter choices, and we compare the results with the case of a system comprising only normal leads. Then we study the infinite-U Anderson model with a superconducting lead at zero temperature, and we discuss the effect of the proximity of the superconductor on the pumped charge.

PACS numbers: 73.23.-b, 74.45.+c

I. INTRODUCTION

A finite charge can be pumped through a mesoscopic system in the absence of an applied bias voltage by changing periodically in time some parameters of the system. If the parameters are changed in time slowly with respect to the lifetime of the electrons in the system, pumping is *adiabatic*. The idea of electron pumping through a mesoscopic system is based on a work of Thouless.¹ Since then a large number of theoretical^{2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19} and experimental^{20,21,22,23} works has been dedicated to this field. Adiabatic pumping has a geometric nature: The pumped charge depends only on the area spanned by the pumping cycle in parameter space and not on its detailed timing. It can therefore be related to geometric phases.^{7,8,9,10} In normal conductors transport is due to the transfer of individual quasiparticles. In superconducting devices, quasiparticle transport is hindered by the gap, and charge pumping is due to Cooper pairs.^{10,11,12,13} In hybrid normal-superconducting systems, Andreev reflection at the interface between the normal and the superconducting region allows for sub-gap charge transport in the system. In the absence of Coulomb interaction, adiabatic pumping in such hybrid systems can be studied by generalizing in Nambu space the scattering approach to adiabatic pumping.^{14,15,16,17}

In the present paper we study a system consisting of a quantum-dot attached to a normal conducting lead and a superconducting lead (N-dot-S system) as shown

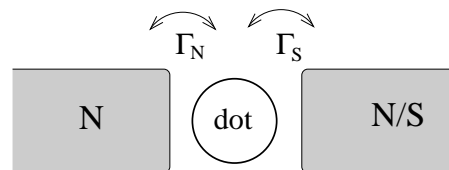


FIG. 1: Sketch of the dot attached to two leads. We are interested in the N-dot-S system, where the left lead is normal conducting and the right lead is superconducting and will compare to the N-dot-N system, where both leads are normal conducting. Left and right lead are attached to the dot via tunnel barriers. The tunneling strength is described by the respective rate Γ_N and Γ_S . Possible time-dependent parameters are: the strength of the tunnel couplings Γ_N and Γ_S , and the dot-level position ϵ .

in Fig. 1. In the absence of Coulomb repulsion in the dot, adiabatic pumping through such a system has been discussed in Refs. 14,15,16. On the other hand, when Coulomb repulsion is present, only DC transport through the N-dot-S system has been investigated.^{24,25,26,27,28,29}

We generalize a recently developed formalism to calculate the pumped charge through an interacting quantum dot^{18,19} to the case of the N-dot-S system. First, we apply this formalism to a non-interacting quantum dot, studying systematically the ratio between the charge pumped through the N-dot-S system and the charge pumped through a system consisting of a dot attached to two normal conducting leads (N-dot-N system). This ra-

tio was calculated in Refs. 14,15,16 for a non-interacting resonant system at zero temperature. We consider finite temperature and extend the analysis to the off-resonant situation. Finally, we apply the formalism to the infinite-U Anderson model in the Kondo regime. We discuss the pumped charge as a function of the level position and the superconducting gap and compare to the interacting N-dot-N system.

II. MODEL AND FORMALISM

In this paper we consider a quantum dot tunnel coupled to one normal and one superconducting lead, as shown in Fig. 1. The system is described by the Hamiltonian: $H = \sum_{\alpha=N,S} (H_{\alpha} + H_{\text{tunn},\alpha}) + H_{\text{dot}}$. The dot Hamiltonian reads $H_{\text{dot}} = \sum_{\sigma} \epsilon(t) d_{\sigma}^{\dagger} d_{\sigma} + U n_{\uparrow} n_{\downarrow}$, where σ is a spin label, $n_{\sigma} = d_{\sigma}^{\dagger} d_{\sigma}$ and we allow the dot level ϵ to depend on time t . The Hamiltonian for the normal lead reads $H_N = \sum_{k,\sigma} \epsilon_k c_{N,k\sigma}^{\dagger} c_{N,k\sigma}$, and the one for the superconducting lead reads $H_S = \sum_{k,\sigma} \epsilon_k c_{S,k\sigma}^{\dagger} c_{S,k\sigma} + \sum_k (\Delta c_{S,k\uparrow}^{\dagger} c_{S,-k\downarrow}^{\dagger} + H.c.)$, with $\Delta \in \mathbb{R}$ being the su-

perconducting order parameter. We take Δ to be k -independent, which is appropriate for conventional BCS superconductors such as aluminum and niobium. The operator d_{σ}^{\dagger} (d_{σ}) creates (annihilates) a dot electron with spin σ , the operator $c_{\alpha,k\sigma}^{\dagger}$ ($c_{\alpha,k\sigma}$) creates (annihilates) an electron in lead α with momentum k and spin σ . The tunnel coupling between the dot and the leads is taken into account by the tunneling Hamiltonians $H_{\text{tunn},\alpha} = \sum_{k,\sigma} (V_{\alpha}(t) c_{\alpha,k\sigma}^{\dagger} d_{\sigma} + H.c.)$, where $\alpha = N,S$, and the tunneling amplitudes V_{α} can be time-dependent. We assume that the phase of V_{α} does not depend on time, since a time-dependent phase would describe an applied bias voltage. From now on we work in Nambu space, defining the fields: $\Psi_{\alpha,k} = (c_{\alpha,k\uparrow}, c_{\alpha,-k\downarrow}^{\dagger})$, and $\Phi = (d_{\uparrow}, d_{\downarrow}^{\dagger})$.

Generalizing the approach of Refs. 18,19, the pumped current can be related to the local Green's function of the dot. In particular, the first adiabatic correction to the charge current from the dot to the left (normal) conductor through the barrier is given by

$$J(t) = -\frac{e}{2\pi} \int d\omega \left(-\frac{\partial f}{\partial \omega} \right) \text{Re} \left\{ \text{Tr} \left[\tau_3 \frac{d}{dt} \left[\Gamma_N(t) \hat{G}_0^r(\omega, t) \right] \left(\hat{G}_0^r(\omega, t) \right)^{-1} \hat{G}_0^a(\omega, t) \right] \right\} + J^{\text{corr}}, \quad (1)$$

where $f(\omega)$ is the Fermi function, the caret indicates a matrix in Nambu space and τ_3 is the Pauli matrix given by $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The first term in Eq. (1) corresponds to the so called average-time approximation,¹⁸ while J^{corr} (discussed below) contains contributions due to vertex corrections¹⁹ and it is zero for all cases studied in this paper. The instantaneous retarded Green's function is defined as $\hat{G}_0^r(\tau, 0, t) = -i\Theta(\tau) \langle \{ \Phi^{\dagger}(\tau), \Phi(0) \} \rangle$, and its Fourier transform is $\hat{G}_0^r(\omega, t) = \int d\tau e^{i\omega\tau} \hat{G}_0^r(\tau, t)$. The last argument (t) of the Green's function is the time with respect to which the adiabatic expansion has been performed.¹⁸ In particular, the instantaneous retarded Green's function, $\hat{G}_0^r(\omega, t)$ is computed with all time-dependent parameters frozen at time t . The advanced Green's function is related to the retarded one by $\hat{G}_0^a(\omega, t) = (\hat{G}_0^r(\omega, t))^{\dagger}$. The intrinsic line width Γ_{α} is defined as $\Gamma_{\alpha}(t, t') = 2\pi\rho_{\alpha} V_{\alpha}(t) V_{\alpha}^*(t')$ with $\alpha = N, S$ and $\Gamma_{\alpha}(t) = \Gamma_{\alpha}(t, t)$. The normal lead is supposed to have flat bands with a constant density of states ρ_N . The density of states ρ_S is the one of the superconducting lead in its normal state and it is constant, too.

We now discuss the average-time approximation and give an expression for the correction term J^{corr} . The average time approximation neglects terms due to vertex corrections¹⁹ in evaluating the adiabatic expan-

sion of the self energy. The self energy is defined by the Dyson equation $G(t', t'', t) = g(t', t'', t) + \int dt_1 \int dt_2 G(t', t_1, t) \Sigma(t_1, t_2, t) g(t_2, t'', t)$ and it is a functional of the time-dependent Hamiltonian $H(\tau)$, where $\tau \in [t_1, t_2]$. The adiabatic expansion is performed by linearizing the time-dependence of the Hamiltonian around a fixed time t (denoted as a third argument of the self energy). In order to solve the Dyson equation for the adiabatic expansion of the Green's function, the average-time approximation is performed on top of the adiabatic expansion. Such approximation consists in replacing the linear time dependence on τ by the average time $(t_1 + t_2)/2$ of the interval $[t_1, t_2]$ and thus it neglects the term

$$\Sigma^{\text{corr}}(t_1, t_2, t) = T_K \int_K d\tau \frac{\delta \Sigma}{\delta H(\tau)} \Big|_{H(\tau)=H(t)} \left(\tau - \frac{t_1 + t_2}{2} \right) \dot{H}_{\tau}(t),$$

where the time integral is calculated along the Keldysh contour, T_K is the time-ordering operator along the Keldysh contour, and $\delta \Sigma / \delta H(\tau)$ a functional derivative. Finally, the correction to the pumped current can be

written as

$$J^{\text{corr}} = \frac{e}{2\pi} \int d\omega \int \frac{d\omega'}{\pi} \text{ReTr} \left[\hat{G}_0^a(\omega', t) \tau_3 \hat{\Gamma}_N(\omega, t) \hat{G}_0^r(\omega', t) \right. \\ \left. \frac{\hat{\Sigma}^{\text{corr}, <}(\omega', t) + f(\omega') \left(\hat{\Sigma}^{\text{corr}, r}(\omega', t) - \hat{\Sigma}^{\text{corr}, a}(\omega', t) \right)}{\omega' - \omega - i0_+} \right]. \quad (2)$$

It is important to point out that the average-time approximation is exact whenever: i) the dot is non-interacting, ii) temperature is zero, the interaction is arbitrary, and the system can be effectively mapped to a Fermi liquid, and iii) when interaction is infinite and the self-energy Σ is calculated up to linear order in Γ . Therefore the correction term equals zero in all cases discussed in this paper.

In the following we are interested in calculating the charge pumped through the system in the weak pumping regime (bilinear response in the pumping fields³⁰). The pumped charge Q is related to the pumped current by $Q = \int_0^T J(t) dt$, where T is the period of the cycle. We can choose as pumping parameters, $X(t)$ and $Y(t)$, any two of the three quantities $\epsilon(t) = \bar{\epsilon} + \delta\epsilon(t)$, $\Gamma_S(t) = \bar{\Gamma}_S + \delta\Gamma_S(t)$ and $\Gamma_N(t) = \bar{\Gamma}_N + \delta\Gamma_N(t)$. The time-averaged part \bar{X} is denoted by a bar and the time-dependent part by $\delta X(t)$. For the pumped charge, with pumping parameters indicated in the subscript, we find

$$Q_{X,Y} = -\frac{e\eta(X,Y)}{4\pi} \int d\omega \left(-\frac{\partial f}{\partial \omega} \right) \text{ReTr} \tau_3 \\ \left\{ \frac{\partial}{\partial \bar{X}} \left((\bar{\Gamma}_N - \bar{\Gamma}_S) \bar{G}_0^r \right) \frac{\partial}{\partial \bar{Y}} \left((\bar{G}_0^r)^{-1} \bar{G}_0^a \right) \right. \\ \left. - \frac{\partial}{\partial \bar{Y}} \left((\bar{\Gamma}_N - \bar{\Gamma}_S) \bar{G}_0^r \right) \frac{\partial}{\partial \bar{X}} \left((\bar{G}_0^r)^{-1} \bar{G}_0^a \right) \right\}. \quad (3)$$

The prefactor $\eta(X, Y)$ accounts for the amplitude and the relative phase of the pumping parameters and is equal to the surface in parameter space enclosed in one pumping cycle. It is defined as $\eta(X, Y) = \int_0^T d\tau \left[\frac{d}{d\tau} \delta X(\tau) \right] \delta Y(\tau)$. The order of the parameters in the argument of η is important and has to be respected in the formulas above (changing their order results in an additional minus sign.) \bar{G}_0^r and \bar{G}_0^a are the retarded and advanced dot Green's functions (both matrices in Nambu space although the caret has been omitted to simplify the notation) with all parameters taken at their time-averaged value.

III. NONINTERACTING QUANTUM DOT

We start by considering the simple case of a non-interacting quantum dot. The results which we obtain for the non-interacting dot, can equivalently be calculated by a generalization of Brouwer's formula to the N-dot-S system.^{14,15,16} The instantaneous Green's function

of the dot is

$$\hat{G}^r(\omega) = \left(\omega - \epsilon\tau_3 + \frac{1}{2}i\Gamma_N - \Sigma_S \right)^{-1}, \quad (4)$$

where the self energy due to the proximity of the superconducting lead reads

$$\hat{\Sigma}_S = \begin{pmatrix} -\frac{\Gamma_S}{2} \frac{\omega}{\sqrt{\Delta^2 - \omega^2}} & \frac{\Gamma_S}{2} \frac{\Delta}{\sqrt{\Delta^2 - \omega^2}} \\ \frac{\Gamma_S}{2} \frac{\Delta}{\sqrt{\Delta^2 - \omega^2}} & -\frac{\Gamma_S}{2} \frac{\omega}{\sqrt{\Delta^2 - \omega^2}} \end{pmatrix}. \quad (5)$$

We now focus on the limit of large superconducting gap $\Delta \rightarrow \infty$, when $(\hat{\Sigma}_S)_{1,2} = (\hat{\Sigma}_S)_{2,1} = \Gamma_S/2$ and $(\hat{\Sigma}_S)_{1,1} = (\hat{\Sigma}_S)_{2,2} = 0$.

Let us consider first the particle-particle spectral function, which is defined as $A_{1,1}(\omega) = -(1/\pi) \text{Im} \{ G_{1,1}^r \}$. The spectral function conveys important information on the proximity effect in the quantum dot. Furthermore, the DC transport properties are influenced by the energy distribution of spectral weight, for example the Andreev-reflection probability $R_A(\omega)$ has a similar structure as $A_{1,1}(\omega)$. Due to the close relation of the spectral function with the retarded Green's function appearing in the formula for the pumped charge, the spectral function will also prove to be a useful quantity to analyze pumping in the N-dot-S system. It turns out that $A_{1,1}$ is given by a linear combination of two Lorentzians and reads

$$A_{1,1}(\omega) = \frac{1}{2} \left(1 + \frac{\epsilon}{\sqrt{\frac{\Gamma_S^2}{4} + \epsilon^2}} \right) L_{\Gamma_N} \left(\omega - \sqrt{\frac{\Gamma_S^2}{4} + \epsilon^2} \right) \\ + \frac{1}{2} \left(1 - \frac{\epsilon}{\sqrt{\frac{\Gamma_S^2}{4} + \epsilon^2}} \right) L_{\Gamma_N} \left(\omega + \sqrt{\frac{\Gamma_S^2}{4} + \epsilon^2} \right), \quad (6)$$

with the Lorentzian bell defined by $L_{\Gamma}(\omega - E) = (\frac{\Gamma}{2\pi}) / ((\omega - E)^2 + (\frac{\Gamma}{2})^2)$. The presence of two peaks in the particle-particle spectral function [Eq. (6)] is a signature of the proximity effect induced by the superconductor. The width of the Lorentzians appearing in Eq. (6) is equal to the coupling to the normal lead, while their positions are shifted, with respect to the dot-level position, by the coupling to the superconducting lead Γ_S . This fact can be referred to as a *proximization* of the dot-level position, which might be contrasted to the level *renormalization* due to Coulomb interaction. The repulsion of the two Lorentzian peaks with increasing Γ_S reflects the increasing coupling between electron and hole excitations in the dot [represented by the off-diagonal terms in Eq. (5)].

In the non-interacting case Eq. (3) can be written in a compact form if the two pumping parameters, denoted by $X(t)$ and $Y(t)$, are chosen among the three quantities $\{\frac{\Gamma_N}{2}(t), \frac{\Gamma_S}{2}(t), \epsilon(t)\}$.³¹ We denote by Z the parameter which is kept constant in time. Eq. (3) simplifies to

$$Q_{X,Y} = \frac{4e}{\pi} \eta(X, Y) \varepsilon_{X,Y,Z} \\ \int d\omega \left(-\frac{\partial f}{\partial \omega} \right) \frac{\bar{\Gamma}_S}{2} \left(\frac{\bar{\Gamma}_N}{2} \right)^2 \frac{\partial}{\partial Z} \left(\frac{1}{N} \right), \quad (7)$$

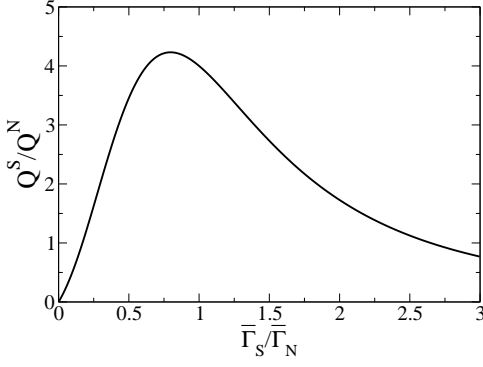


FIG. 2: Ratio between the pumped charge in the N-dot-S and in the N-dot-N systems, $Q_{X,Y}^S/Q_{X,Y}^N$, as a function of $\bar{\Gamma}_S/\bar{\Gamma}_N$. The curve is the same for any couple of pumping parameters X, Y . Temperature is equal to zero and the bare dot level is at resonance.

where N is the denominator of the particle-particle spectral function with time-averaged parameters

$$N = \left\{ \omega^2 - \bar{\epsilon}^2 - \left[\left(\frac{\bar{\Gamma}_S}{2} \right)^2 - \left(\frac{\bar{\Gamma}_N}{2} \right)^2 \right] \right\}^2 + 4 \left(\frac{\bar{\Gamma}_N}{2} \right)^2 \left[\left(\frac{\bar{\Gamma}_S}{2} \right)^2 + \bar{\epsilon}^2 \right]. \quad (8)$$

The quantity $\eta(X, Y)$ is the surface in parameter space enclosed in one pumping cycle. The antisymmetric tensor $\varepsilon_{X,Y,Z}$ has to be evaluated respecting the order of the parameters $\{\frac{\Gamma_N}{2}(t), \frac{\Gamma_S}{2}(t), \epsilon(t)\}$. In the case of weak-pumping with the two tunnel-barrier strengths, i.e $X(t), Y(t) = \Gamma_{S,N} + \delta\Gamma_{S,N}(t)$, and $\bar{Z} = \bar{\epsilon}$, we can rewrite Eq. (7) as

$$Q_{\Gamma_N, \Gamma_S} = \frac{e}{\pi} \eta(\Gamma_N, \Gamma_S) \frac{1}{2\Gamma_S} \int d\omega \left(-\frac{\partial f}{\partial \omega} \right) \frac{\partial \bar{R}_A}{\partial \bar{\epsilon}}, \quad (9)$$

which relates the pumped charge to the probability of Andreev reflection \bar{R}_A , computed with the time-dependent parameters frozen at their average value. The probability of Andreev reflection is given by³²

$$\begin{aligned} R_A &= \frac{\Gamma_N^2 \Gamma_S^2}{4 \left[\omega^2 - \epsilon^2 + \frac{1}{4} (\Gamma_N^2 - \Gamma_S^2) \right]^2 + \Gamma_N^2 (\Gamma_S^2 + 4\epsilon^2)} \\ &= \frac{\pi}{4} \frac{\Gamma_N \Gamma_S^2}{\frac{\Gamma_S^2}{4} + \frac{\Gamma_N^2}{4} + \epsilon^2 + \omega^2} \left[L_{\Gamma_N} \left(\omega - \sqrt{\frac{\Gamma_S^2}{4} + \epsilon^2} \right) \right. \\ &\quad \left. + L_{\Gamma_N} \left(\omega + \sqrt{\frac{\Gamma_S^2}{4} + \epsilon^2} \right) \right]. \end{aligned} \quad (10)$$

We are now interested in comparing our results to the ones obtained for an N-dot-N system (see Fig. 1 with normal-conducting right lead). The current pumped

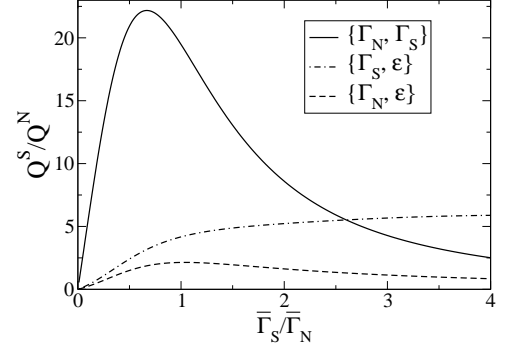


FIG. 3: Ratio between the pumped charge in N-dot-S and in the N-dot-N system, $Q_{X,Y}^S/Q_{X,Y}^N$, as a function of $\bar{\Gamma}_S/\bar{\Gamma}_N$. Temperature is equal to $k_B T = 1.5 \bar{\Gamma}_N$. The pumping parameters are Γ_N and Γ_S (solid line), Γ_N and ϵ (dashed line) and Γ_S and ϵ (dotted and dashed line). The bare dot level is at resonance.

through a non-interacting dot attached to two normal conducting leads can be obtained using Brouwer's formula.² For the case where the tunneling rates $\Gamma_N(t)$ and $\Gamma_S(t)$ are the pumping parameters, it has been found that the current in the presence of one superconducting lead is about a factor 4 bigger than in the case of two normal leads, if the dot level is at resonance ($\epsilon = 0$) and temperature is zero.^{14,15,16} We find exactly the same behavior for all three choices of pumping parameters, at zero temperature. It turns out that the ratio $Q_{X,Y}^S/Q_{X,Y}^N$ (equal for any pairs of pumping parameters) has a maximum, as a function of the level position, at resonance.³³ We plot in Fig. 2 the ratio $Q_{X,Y}^S/Q_{X,Y}^N$, for $\epsilon = 0$, as a function of $\bar{\Gamma}_S/\bar{\Gamma}_N$: a maximum is present when $\bar{\Gamma}_S/\bar{\Gamma}_N \simeq 0.9$. In the limit of small and large $\bar{\Gamma}_S/\bar{\Gamma}_N$ the charge ratio goes to zero. This happens because, in the presence of a superconducting lead, pumping is mediated by the Andreev reflection probability, Eq. (10). On one hand, for $\bar{\Gamma}_S/\bar{\Gamma}_N \ll 1$, the ratio is suppressed since R_A implies higher-order tunneling processes to the superconducting lead with respect to tunneling to the normal lead. On the other hand, for $\bar{\Gamma}_S/\bar{\Gamma}_N \gg 1$, the Lorentzian contributions to R_A get narrower with decreasing $\bar{\Gamma}_N$ and more distant with increasing $\bar{\Gamma}_S$, thus suppressing the Andreev reflection probability at $\omega = 0$ (relevant for zero temperature). In the case of finite temperature the charge ratio depends on the parameter choice and can take values much bigger than 4. Fig. 3 shows the ratio $Q_{X,Y}^S/Q_{X,Y}^N$ for the three different parameter choices at temperature $k_B T = 1.5 \bar{\Gamma}_N$, with the dot level being at resonance.

In the following we discuss some qualitative features, in the high-temperature limit ($k_B T \gg \Gamma_N$), of the ratio $Q_{X,Y}^S/Q_{X,Y}^N$. Expressions for the pumped charge are reported in Appendix A. While for the N-dot-S system the leading order of the pumped charge is the first derivative of the Fermi function, in the N-dot-N system this is the case only when the level position is one of the pumping parameters. The first contribution to Q_{Γ_N, Γ_S}^N starts

with the second derivative of the Fermi function. On the other hand, the parameter Γ_S enters in the spectral density of the N-dot-S system via the proximized peak positions. The high-temperature limit of the charge ratio as a function of temperature is therefore a constant if $\{\Gamma_\alpha, \epsilon\}$ are the pumping parameters and it is quadratic in temperature in the case that $\{\Gamma_N, \Gamma_S\}$ are the pumping parameters:

$$\frac{Q_{\Gamma_N, \Gamma_S}^S}{Q_{\Gamma_N, \Gamma_S}^N} \simeq (k_B T)^2 \frac{\bar{\Gamma}_N \bar{\Gamma}_S}{\left(\bar{\epsilon}^2 + \frac{\bar{\Gamma}_N^2}{4} + \frac{\bar{\Gamma}_S^2}{4}\right)^2} \quad (11a)$$

$$\frac{Q_{\Gamma_N, \epsilon}^S}{Q_{\Gamma_N, \epsilon}^N} \simeq \frac{1}{8} \frac{\bar{\Gamma}_N \bar{\Gamma}_S (\bar{\Gamma}_N + \bar{\Gamma}_S)^2}{\left(\bar{\epsilon}^2 + \frac{\bar{\Gamma}_N^2}{4} + \frac{\bar{\Gamma}_S^2}{4}\right)^2} \quad (11b)$$

$$\frac{Q_{\Gamma_S, \epsilon}^S}{Q_{\Gamma_S, \epsilon}^N} \simeq \frac{1}{8} \frac{\bar{\Gamma}_N \bar{\Gamma}_S (\bar{\Gamma}_N + \bar{\Gamma}_S)^2}{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_N^2}{4} + \frac{\bar{\Gamma}_S^2}{4}} \left(\frac{1}{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_N^2}{4} + \frac{\bar{\Gamma}_S^2}{4}} + \frac{2}{\bar{\Gamma}_N^2} \right). \quad (11c)$$

For all three cases described by Eqs. (11), the ratio of the pumped charge, as a function of the average dot level position, has a maximum when the average dot level is at resonance. At $\bar{\epsilon} = 0$, with $\bar{\Gamma}_S = \bar{\Gamma}_N$, we find that $Q_{\Gamma_N, \epsilon}^S / Q_{\Gamma_N, \epsilon}^N \simeq 2$ and $Q_{\Gamma_S, \epsilon}^S / Q_{\Gamma_S, \epsilon}^N \simeq 4$. Furthermore we note that due to the second term in Eq. (11c), the ratio $Q_{\Gamma_S, \epsilon}^S / Q_{\Gamma_S, \epsilon}^N$ is increasing in a monotonic way for increasing $\bar{\Gamma}_S / \bar{\Gamma}_N$, while $Q_{\Gamma_N, \Gamma_S}^S / Q_{\Gamma_N, \Gamma_S}^N$ and $Q_{\Gamma_N, \epsilon}^S / Q_{\Gamma_N, \epsilon}^N$ have a maximum as a function of $\bar{\Gamma}_S / \bar{\Gamma}_N$. This behavior is present in Fig. 3, where we also observe a large value of the maximum of $Q_{\Gamma_N, \Gamma_S}^S / Q_{\Gamma_N, \Gamma_S}^N$, which is due to the quadratic dependence on $k_B T$.

The plots in Fig. 4 show the pumped charge, as a function of the average dot level position, through the N-dot-S system [panel (a)] and through the N-dot-N system [panel (b)] for different choices of pumping-parameter pairs at $k_B T = \bar{\Gamma}_N$ and for $\bar{\Gamma}_N = \bar{\Gamma}_S$. We point out several differences: In the N-dot-N case the pumped charge is the same for pumping with $\{\Gamma_N, \epsilon\}$ and with $\{\Gamma_S, \epsilon\}$. In the presence of a superconducting lead the symmetry between the two leads is absent as soon as temperature differs from zero. Furthermore we report that the width of the pumped charge is strongly temperature dependent for the N-dot-N system, while for the N-dot-S system the width is saturating for high temperatures to a value that depends solely on the coupling to the leads, Γ_N and Γ_S . This is due to the fact that the gap of the superconductor is always bigger than temperature and no quasiparticle transport, which is sensitive to temperature, takes place between the superconductor and the dot. The symmetry of the pumped charge as a function of the dot-level position is present for both systems. For the N-dot-S system, when pumping with Γ_N and Γ_S , this happens despite the fact that, at least at high temperatures, pumping is dominated by the proximized level position.

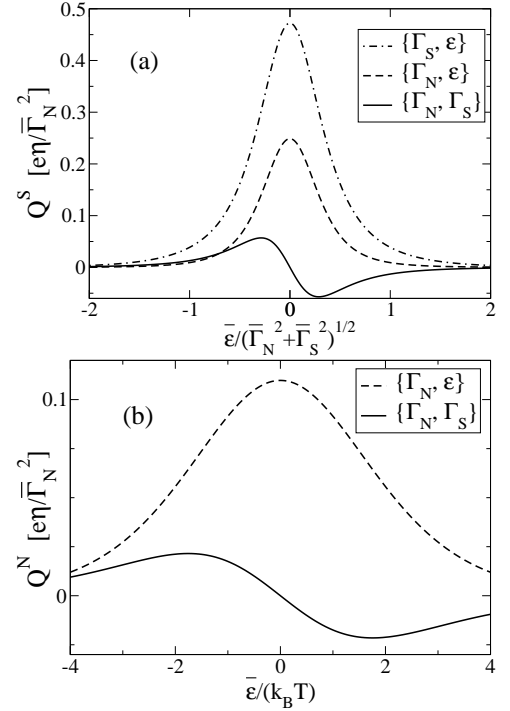


FIG. 4: Pumped charge in the N-dot-S [panel (a)] and in the N-dot-N [panel (b)] systems as a function of the average level position $\bar{\epsilon}$. Temperature is set to $k_B T = \bar{\Gamma}_N$ and $\bar{\Gamma}_N = \bar{\Gamma}_S$. The pumping parameters are $\{\Gamma_N, \Gamma_S\}$ (solid line), $\{\Gamma_N, \epsilon\}$ (dashed line) and $\{\Gamma_S, \epsilon\}$ (dashed-dotted line). The units on the x-axes are the ones determining the width of the respective function.

It might be interesting to contrast the pumped charge in the N-dot-S system with the DC linear conductance of the same system (here all parameters are time-independent):

$$G = \frac{4e^2}{h} \int d\omega \left(-\frac{\partial f}{\partial \omega} \right) R_A(\omega). \quad (12)$$

In the high temperature limit, i.e. $k_B T \gg \Gamma_N$, the conductance is given by

$$G = -\frac{4e^2}{h} \frac{\pi}{4} \frac{\Gamma_N \Gamma_S^2}{\frac{\Gamma_S^2}{4} + \frac{\Gamma_N^2}{4} + \epsilon^2} \frac{\partial f}{\partial \omega} \left(\sqrt{\frac{\Gamma_S^2}{4} + \epsilon^2} \right). \quad (13)$$

IV. STRONG INTERACTION LIMIT

In the regime of strong Coulomb repulsion double occupation is forbidden and coherent Andreev scattering can occur only if Δ is finite (electrons can enter the superconductor within a time window of order \hbar/Δ). In the Kondo limit, when temperature is zero, we can evaluate the retarded equilibrium dot Green's function making use of the mean-field slave-boson technique.^{26,34} As the dot can only be empty or singly occupied we can describe the singly-occupied dot state by the fermion operator f_σ

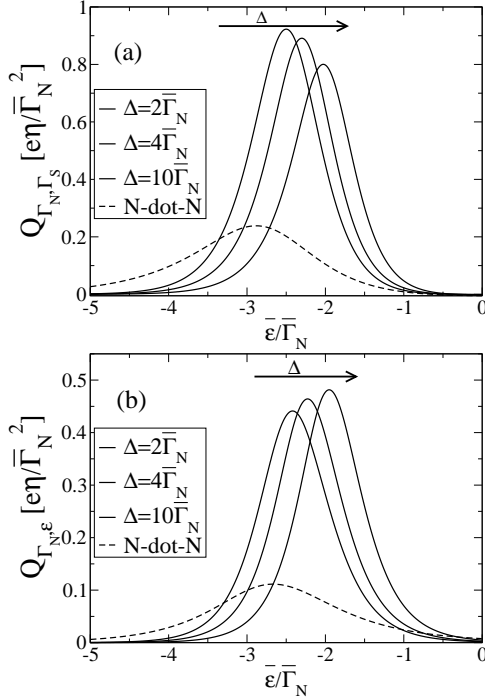


FIG. 5: Pumped charge Q_{Γ_N, Γ_S}^S [panel (a)] and $Q_{\Gamma_N, \epsilon}^S$ [panel (b)] as a function of the average level position for different values of Δ , solid lines, and Q_{Γ_N, Γ_S}^N and $Q_{\Gamma_N, \epsilon}^N$, dashed lines. Temperature is zero and we consider symmetric barriers ($\bar{\Gamma}_N = \bar{\Gamma}_S$). The cut-off energy is $W = 20\bar{\Gamma}_N$.

(known as pseudo fermion) and the empty dot by the boson operator b . The real dot operator d_σ is simply given by $d_\sigma = b^\dagger f_\sigma$. The operators b and f_σ need to fulfill the constraint $b^\dagger b + \sum_\sigma f_\sigma^\dagger f_\sigma = 1$. Such a constraint is introduced in the Hamiltonian using the Lagrange multiplier λ which plays the role of a chemical potential. In the mean-field limit, the boson operator b is replaced by a real number b_0 . The quantities b_0 and λ are determined by minimizing the free energy. Note that we are allowed to use an equilibrium formalism, since after performing the adiabatic approximation we are left with a current formula, Eq. (1), that contains only equilibrium quantities.

Details of the derivation of the retarded dot Green's function by means of the slave-boson technique, in the limit $\Delta \gg \Gamma_S$, are given in Appendix B. It is important to notice that the pumped charge in the deep Kondo regime is zero. Therefore, in the following, we will be interested in the mixed-valence regime only. The pumped charge in the weak-pumping limit for the aforementioned three pairs of pumping parameters can be calculated using Eq. (3). The pumped charge has a similar structure in all three cases. We find, in particular, the general relation

$$-\frac{1}{\eta(\Gamma_N, \epsilon)} \frac{1}{\bar{\Gamma}_S} Q_{\Gamma_N, \epsilon}^S = \frac{1}{\eta(\Gamma_S, \epsilon)} \frac{1}{\bar{\Gamma}_N} Q_{\Gamma_S, \epsilon}^S, \quad (14)$$

which is also valid for the noninteracting N-dot-S system

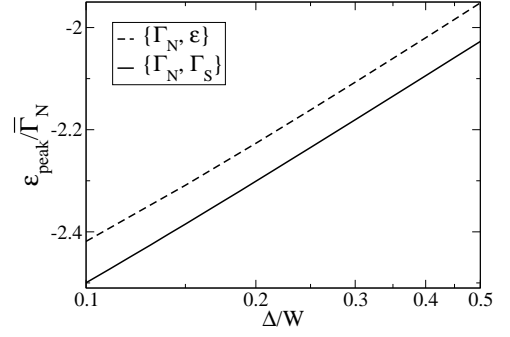


FIG. 6: The peak position ϵ_{peak} is shown as a function of the superconducting gap for different pairs of pumping parameters on a logarithmic scale. The pumping parameters are $\{\Gamma_N, \Gamma_S\}$ (solid line) and $\{\Gamma_N, \epsilon\}$ (dashed line). The temperature is zero and $\bar{\Gamma}_N = \bar{\Gamma}_S = 0.05W$.

at zero temperature.

In analyzing the properties of the pumped charge as a function of the different system parameters we can profit from the fact that, in the Kondo regime, the system behaves like a non-interacting system with renormalized parameters, denoted as $\tilde{\Gamma}_N$, $\tilde{\Gamma}_S$ and $\tilde{\epsilon}$. We should however keep in mind that the renormalized parameters depend in a complex way on all the bare parameters. Panel (a) of Fig. 5 shows the pumped charge Q_{Γ_N, Γ_S}^S (solid lines) as a function of the level position, for increasing values of the superconducting gap Δ . The charge Q_{Γ_N, Γ_S}^N in the N-dot-N is also shown for reference (dashed line). In panel (b), the pumped charge is shown for Γ_N and ϵ chosen as pumping parameters. We note that for both the N-dot-N system and the N-dot-S system (for any value of the superconducting gap Δ) the charge has a bell-like structure. The width of the bell-like curves reflects the width of the respective particle-particle spectral functions, which for the N-dot-S system is given by $\tilde{\Gamma}_N$ while for the N-dot-N system it is given by $\tilde{\Gamma}_N + \tilde{\Gamma}_S$. For the values of parameters considered in Fig. 5 ($\bar{\Gamma}_N = \bar{\Gamma}_S = 0.05W$), the curve relative to $\Delta = 2\bar{\Gamma}_N$ is still very far away from the curve relative to the N-dot-N system.

The striking similarity of the pumped charge for the two different pairs of pumping parameters [compare Figs. 5 (a) and (b) and Eq. (14)] can be explained by the fact that pumping takes place via renormalized parameters. Indeed, in the zero-temperature regime with strong Coulomb interaction, the Green's function of the system depends on the bare parameters solely via renormalized parameters $\tilde{\Gamma}_N$, $\tilde{\Gamma}_S$ and $\tilde{\epsilon}$. Therefore, the detailed behavior of the pumped charge is determined by the derivatives of the renormalized parameters with respect to the bare ones. In particular when the bare dot level is deep below the Fermi energy, i.e. in the Kondo regime, the renormalized parameters are independent of the bare parameters and therefore the pumped charge is zero.

Now, we focus our attention on the dependence of the pumped charge on the superconducting gap. In a non-interacting dot, the pumped charge is always independent

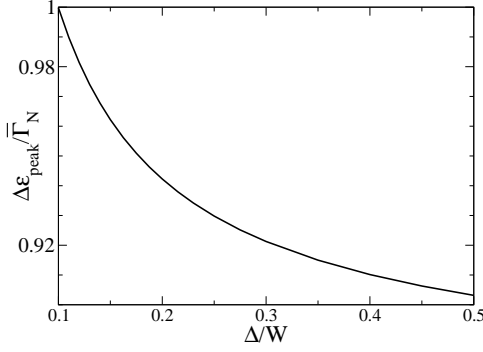


FIG. 7: Peak width $\Delta\epsilon_{\text{peak}}$ for the pumped charge plotted as function of the superconducting gap Δ . The pumping parameters are $\{\Gamma_N, \Gamma_S\}$. The temperature is zero and $\bar{\Gamma}_N = \bar{\Gamma}_S = 0.05W$.

of the superconducting gap Δ at $T = 0$, since the Green's function of Eq. (4), for $\omega = 0$, is independent of Δ . The situation is different in the presence of Coulomb interaction, where the pumped charge depends on the superconducting gap via the renormalized parameters. Now, we investigate how the features of the bell curves, describing the pumped charge as a function of the average level $\bar{\epsilon}$, depend on the gap Δ .

Peak position. Fig. 6 shows that the position of the peak of the pumped charge Q^S depends on $\ln\Delta$. The peak position is shifted towards zero (from below) for increasing gap. This is consistent with the fact that the renormalized level acquires a logarithmic correction in Δ proportional to Γ_S .²⁶

Peak width. Fig. 7 shows the peak width $\Delta\epsilon_{\text{peak}}$ (half-height-width) for the pumped charge $Q^S_{\Gamma_N, \Gamma_S}$ as a function of the superconducting gap Δ . As stated before, the peak width has an analogous behavior to the width of the spectral function. The coupling to the superconducting lead influences the width of the spectral function via the renormalized coupling $\bar{\Gamma}_N$. The decrease of $\bar{\Gamma}_N$ with increasing superconducting gap induces the Δ -dependence shown in Fig. 7.

V. CONCLUSIONS

We have extended a Green's function formalism to compute the pumped current through a quantum dot

with Coulomb interaction, to the case when the dot is tunnel-coupled to one normal and one superconducting lead. First, we have applied our formalism to a non-interacting dot in an N-dot-S configuration, where we derived a compact formula for the pumped charge, and we calculated the ratio between the charge pumped through the N-dot-S system and through an N-dot-N system. At finite temperature, we found different characteristic behaviors of this ratio, depending on the choice of pumping parameters. Then, we have studied the pumped charge through a quantum dot with infinitely strong Coulomb interaction at zero temperature using the mean-field slave-boson approach. In particular, we have focused our attention on the influence of the superconducting gap Δ on the pumped charge. We have explained features of the pumped charge exploiting the mapping to a non-interacting system with renormalized parameters. In the end we wish to comment on the experimental realization of the pumping mechanism discussed theoretically in this article. The system depicted in Fig. 1 can be realized in analogy to an experiment reported recently by van Dam *et al.*³⁵. Here a semiconductor quantum dot is contacted with superconductors, using indium arsenide nanowires combined with local gate voltages to create a quantum dot with tunable coupling to the leads. Superconductivity is induced in the wires by means of the proximity effect using aluminum contacts. In order to realize pumping, time-dependent control of the system parameters is required.

APPENDIX A: PUMPED CHARGE IN THE NON-INTERACTING DOT: $k_B T \gg \Gamma_N$

In this appendix we report the expressions for the pumped charge through the non-interacting dot in the weak-pumping regime in the limit $k_B T \gg \Gamma_N$.

N-dot-S system From Eq. (7), for the three possible choices of pumping fields, we obtain:

$$Q^S_{\Gamma_N, \Gamma_S} \simeq e\eta(\Gamma_N, \Gamma_S) \frac{\bar{\epsilon}\bar{\Gamma}_N\bar{\Gamma}_S}{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_N^2}{4} + \frac{\bar{\Gamma}_S^2}{4}} \left[\frac{1}{4} \frac{1}{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_N^2}{4} + \frac{\bar{\Gamma}_S^2}{4}} \frac{\partial f}{\partial \omega} \left(\sqrt{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_S^2}{4}} \right) - \frac{1}{8} \frac{1}{\sqrt{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_S^2}{4}}} \frac{\partial^2 f}{\partial \omega^2} \left(\sqrt{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_S^2}{4}} \right) \right] \quad (\text{A1})$$

$$Q^S_{\Gamma_N, \epsilon} \simeq -e\eta(\Gamma_N, \epsilon) \frac{\bar{\Gamma}_N\bar{\Gamma}_S^2}{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_N^2}{4} + \frac{\bar{\Gamma}_S^2}{4}} \left[\frac{1}{4} \frac{1}{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_N^2}{4} + \frac{\bar{\Gamma}_S^2}{4}} \frac{\partial f}{\partial \omega} \left(\sqrt{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_S^2}{4}} \right) - \frac{1}{8} \frac{1}{\sqrt{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_S^2}{4}}} \frac{\partial^2 f}{\partial \omega^2} \left(\sqrt{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_S^2}{4}} \right) \right] \quad (\text{A2})$$

$$Q_{\Gamma_S, \epsilon}^S \simeq e\eta(\Gamma_S, \epsilon) \frac{\bar{\Gamma}_N^2 \bar{\Gamma}_S}{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_N^2}{4} + \frac{\bar{\Gamma}_S^2}{4}} \left[\left(\frac{1}{4} \frac{1}{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_N^2}{4} + \frac{\bar{\Gamma}_S^2}{4}} + \frac{1}{2\bar{\Gamma}_N^2} \right) \frac{\partial f}{\partial \omega} \left(\sqrt{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_S^2}{4}} \right) - \frac{1}{8} \frac{1}{\sqrt{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_S^2}{4}}} \frac{\partial^2 f}{\partial \omega^2} \left(\sqrt{\bar{\epsilon}^2 + \frac{\bar{\Gamma}_S^2}{4}} \right) \right] \quad (\text{A3})$$

N-dot-N system We compare the results for the N-dot-S system to the charge pumped through the N-dot-N system by varying the respective pumping parameters. We find:

$$Q_{\Gamma_N, \Gamma_S}^N \simeq -\frac{e}{2} \eta(\Gamma_N, \Gamma_S) \frac{\partial^2 f}{\partial \omega^2}(\bar{\epsilon}) \quad (\text{A4})$$

$$Q_{\Gamma_N, \epsilon}^N \simeq -2e\eta(\Gamma_N, \epsilon) \frac{\bar{\Gamma}_S}{(\bar{\Gamma}_N + \bar{\Gamma}_S)^2} \frac{\partial f}{\partial \omega}(\bar{\epsilon}) \quad (\text{A5})$$

$$Q_{\Gamma_S, \epsilon}^N \simeq 2e\eta(\Gamma_S, \epsilon) \frac{\bar{\Gamma}_N}{(\bar{\Gamma}_N + \bar{\Gamma}_S)^2} \frac{\partial f}{\partial \omega}(\bar{\epsilon}). \quad (\text{A6})$$

APPENDIX B: MEAN-FIELD SLAVE-BOSON TECHNIQUE FOR THE N-DOT-S SYSTEM

In this Appendix we report details of the slave-boson method for the N-dot-S system.²⁶ We calculate instantaneous equilibrium Green's functions where all parameters are always taken at their time-averaged value. We therefore omit the bar and, e.g., write Γ_N instead of $\bar{\Gamma}_N$. Furthermore all Green's functions considered in this appendix are 2×2 matrices in Nambu space and we omit the caret.

The free energy of the system can be written as a function of the Matsubara Green's function \mathcal{G} [related to the retarded Green's function by the analytic continuation $(i\omega_n \rightarrow \omega + i\delta)$]:^{36,37}

$$F = -k_B T \sum_{n=-\infty}^{\infty} \text{Tr} \left\{ \ln \tilde{\mathcal{G}}^{-1}(i\omega_n) \right\} + \lambda b_0^2 + \epsilon, \quad (\text{B1})$$

where the trace is performed in Nambu space and $\omega_n = \pi(2n+1)/\beta$ is a fermionic Matsubara frequency. The pseudo-fermion Green's function $\tilde{\mathcal{G}}$ has the same functional form of the noninteracting Matsubara Green's function \mathcal{G} , but contains the renormalized parameters $\tilde{\Gamma}_N = b_0^2 \Gamma_N$, $\tilde{\Gamma}_S = b_0^2 \Gamma_S$ and $\tilde{\epsilon} = \epsilon + \lambda$. Minimizing the free energy with respect to the variables λ and b_0 we find the equations:

$$0 = b_0^2 + k_B T \sum_{n=-\infty}^{\infty} \text{Tr} \left\{ \tilde{\mathcal{G}}(i\omega_n) \sigma_z \right\} \quad (\text{B2a})$$

$$0 = \lambda + k_B T \sum_{n=-\infty}^{\infty} \text{Tr} \left\{ \tilde{\mathcal{G}}(i\omega_n) \Gamma(i\omega_n) \right\}. \quad (\text{B2b})$$

The matrix in Nambu space $\Gamma(i\omega_n)$ is given by

$$\Gamma(i\omega_n) = \begin{pmatrix} -i\frac{\Gamma_N}{2} \text{sign}(\omega_n) & 0 \\ 0 & -i\frac{\Gamma_N}{2} \text{sign}(\omega_n) \end{pmatrix} + \begin{pmatrix} -i\frac{\Gamma_S}{2} \frac{\omega_n}{\sqrt{\omega_n^2 + \Delta^2}} & \frac{\Gamma_S}{2} \frac{\Delta}{\sqrt{\omega_n^2 + \Delta^2}} \\ \frac{\Gamma_S}{2} \frac{\Delta}{\sqrt{\omega_n^2 + \Delta^2}} & -i\frac{\Gamma_S}{2} \frac{\omega_n}{\sqrt{\omega_n^2 + \Delta^2}} \end{pmatrix}. \quad (\text{B3})$$

Equations (B2a) and (b) are used to determine λ and b_0 . At zero temperature we can replace the sum over the Matsubara frequencies by an integral. Furthermore, we consider the limit $\Delta \gg \Gamma_S$ and we perform the following approximation²⁶

$$\Gamma(i\omega_n) \approx \begin{cases} \begin{pmatrix} -i\frac{\Gamma_N}{2} \text{sign}(\omega_n) & \frac{\Gamma_S}{2} \\ \frac{\Gamma_S}{2} & -i\frac{\Gamma_N}{2} \text{sign}(\omega_n) \end{pmatrix} & \text{for } \omega < \Delta \\ \begin{pmatrix} -i\left(\frac{\Gamma_N}{2} + \frac{\Gamma_S}{2}\right) \text{sign}(\omega_n) & 0 \\ 0 & -i\left(\frac{\Gamma_N}{2} + \frac{\Gamma_S}{2}\right) \text{sign}(\omega_n) \end{pmatrix} & \text{for } \omega > \Delta. \end{cases} \quad (\text{B4})$$

Under such an approximation, Eqs. (B2a) and (b) read

$$b_0^2 = \frac{2\tilde{\epsilon}}{\pi\sqrt{\tilde{\epsilon}^2 + \frac{\tilde{\Gamma}_S^2}{4}}} \left[\arctan\left(\frac{\Delta + \frac{\tilde{\Gamma}_N}{2}}{\sqrt{\tilde{\epsilon}^2 + \frac{\tilde{\Gamma}_S^2}{4}}}\right) - \arctan\left(\frac{\frac{\tilde{\Gamma}_N}{2}}{\sqrt{\tilde{\epsilon}^2 + \frac{\tilde{\Gamma}_S^2}{4}}}\right) \right] + \frac{2}{\pi} \left[\arctan\left(\frac{W + \frac{\tilde{\Gamma}_N}{2} + \frac{\tilde{\Gamma}_S}{2}}{\tilde{\epsilon}}\right) - \arctan\left(\frac{\Delta + \frac{\tilde{\Gamma}_N}{2} + \frac{\tilde{\Gamma}_S}{2}}{\tilde{\epsilon}}\right) \right] \quad (\text{B5a})$$

$$\lambda = \frac{\Gamma_N}{2\pi} \ln\left(\frac{\left(\Delta + \frac{\tilde{\Gamma}_N}{2}\right)^2 + \tilde{\epsilon}^2 + \frac{\tilde{\Gamma}_S^2}{4}}{\frac{\tilde{\Gamma}_N^2}{4} + \tilde{\epsilon}^2 + \frac{\tilde{\Gamma}_S^2}{4}}\right) + \frac{\Gamma_N + \Gamma_S}{2\pi} \ln\left(\frac{W^2}{\left(\Delta + \frac{\tilde{\Gamma}_N}{2} + \frac{\tilde{\Gamma}_S}{2}\right)^2 + \tilde{\epsilon}^2}\right) + \frac{\tilde{\Gamma}_S\Gamma_S}{2\pi\sqrt{\frac{\tilde{\Gamma}_S^2}{4} + \tilde{\epsilon}^2}} \left[\arctan\left(\frac{\Delta + \frac{\tilde{\Gamma}_N}{2}}{\sqrt{\tilde{\epsilon}^2 + \frac{\tilde{\Gamma}_S^2}{4}}}\right) - \arctan\left(\frac{\frac{\tilde{\Gamma}_N}{2}}{\sqrt{\tilde{\epsilon}^2 + \frac{\tilde{\Gamma}_S^2}{4}}}\right) \right], \quad (\text{B5b})$$

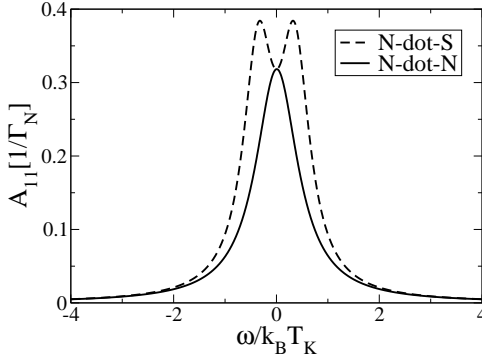


FIG. 8: Spectral density of the dot in the N-dot-N (solid line) and in the N-dot-S system (dashed line) in the Kondo regime ($\epsilon = -0.25W$, $\Gamma_N = \Gamma_S = 0.05W$). The gap of the superconductor is half as big as the bandwidth cut-off ($\Delta = 0.5W$). The Kondo-temperature of the N-dot-N and the N-dot-S system differ by a huge factor: $T_k^N = 7.8 \times 10^{-4}W$ and $T_k^S = 1.3 \times 10^{-6}W$.

where we introduced a cut-off energy W representing the bandwidth of the leads.

Finally, the retarded dot Green's function G^{ret} is obtained from the retarded pseudo-fermion Green's function \tilde{G}^{ret} by means of the relation

$$G^{\text{ret}} = b_0^2 \tilde{G}^{\text{ret}}. \quad (\text{B6})$$

From the retarded Green's function of the N-dot-S system, we can calculate the spectral density $A_{1,1}^S = -(1/\pi)\text{Im}G_{1,1}^{\text{ret}}$ and compare to the one for the N-dot-N system, in the same regime. The result is shown in Fig. 8. We find that the Kondo temperature in the N-dot-S system is much smaller than in the N-dot-N system (note that the width of the two curves is rescaled by the respective Kondo temperature). Nevertheless, in the deep Kondo regime, the spectral weight at the Fermi energy is the same for both systems. The strength of the Kondo-effect is therefore not affected by the presence of the superconductor in the limit of a large but finite gap Δ .

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